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Black hole entropy and topological strings on generalized CY manifolds

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Abstract: The H. Ooguri, A. Strominger and C. Vafa conjecture $Z_{BH} = |Z_{top}|^2$ is extended for the topological strings on generalized CY manifolds. It is argued that the classical black hole entropy is given by the *generalized Hitchin functional*, which defines by critical points a generalized complex structure on X. This geometry differs from an ordinary geometry if $b_1(X) \neq 0$. In a critical point the generalized Hitchin functional equals to Legendre transform of the free energy of generalized topological string. The examples of T^6 and $T^2 \times K3$ are considered in details.

Keywords: Differential and Algebraic Geometry, Black Holes in String Theory, Topological Strings.

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Contents

1. Introduction

In [1] H. Ooguri, A. Strominger and C. Vafa (OSV) suggested a relation between the black hole entropy and the topological string partition function symbolically written as $Z_{BH} = |Z_{top}|^2$. In [2] N. Nekrasov and in [3] R. Dijkgraaf et. al explained that at the classical level the black hole entropy and the topological strings partition function are related to a certain Hitchin functional [4] for real three-forms, which defines by critical points a CY structure on a real six-dimensional manifold X, so $Z_{hit} = Z_{BH} = |Z_{top}|^2$. The important relation between Hitchin functional [4] and the quantization of the topological B-model [5] was shown in [6] by A. Gerasimov and S. Shatashvili. Moreover, in [6] was also suggested to use the generalized Hitchin functional [7], whose degrees of freedom are extended by one-forms and five-forms on X. The necessity to turn on forms of all ranks was proposed in [2] in the perspective of a certain seven-dimensional topological theory. See [8] for topological strings in generalized complex space [7, 9].

The whole construction is about compactifications with (at least) $\mathcal{N} = 2$ supersymmetry. The usual ones are compactifications on Calabi-Yau manifolds. Then one has the A-model and the B-model [10], parameterized by symplectic or complex structures. They are each examples of generalized complex structures [7, 9] and each can be described by a suitable Hitchin functional [4, 7]. The black hole entropy in the supergravity approximation is equal to the Hitchin functional [1 − 3].

At the one-loop level, however, to reproduce the first quantum correction to the Hitchin functional, one needs to use the generalized Hitchin functional, as was shown in [11]. In other words, for Calabi-Yau compactification, at tree level, only the modes of the ordinary Hitchin functional are turned on, but at one-loop level, to get the right quantum correction, one needs to allow the extra fields of the generalized Hitchin functional to run around in the loop.

The present paper is devoted to answering the following question: Is it possible to find a situation in which it is necessary to use the generalized Hitchin functional at tree level in order to reproduce the supergravity approximation to the black hole entropy? This will happen if the extra fields of the generalized Hitchin functional have expectation values at tree level.

In other words, it will happen in the case of a compactification with at least $\mathcal{N} = 2$ supersymmetry that cannot be described as compactification on a complex manifold (B-model) or a symplectic manifold (A-model) – compactification that requires the language of a generalized complex structure. Concretely, this will happen in compactifications of $\mathcal{N} \geq 2$ supersymmetry on a manifold X with $b_1(X)$ nonzero.

There is a consistent supergravity analysis of $\mathcal{N} = 2$ supersymmetric compactifications with $b_1(X)$ nonzero and using generalized complex geometry [12 – 18]. In this paper, we will show that in this situation, the generalized Hitchin functional reproduces the supergravity approximation to the black hole entropy, generalizing the results of OSV for the Kahler case.

However, actual examples of the framework of [13, 12, 14 – 18] are apparently not yet known. To give concrete examples of our calculations, therefore, we will consider the examples of $X = T^6$ and $X = T^2 \times K3$, which certainly do have $b_1(X)$ nonzero. They have more than $\mathcal{N} = 2$ supersymmetry, so it may be the case that at the loop level, they cannot be described by the generalized Hitchin functional (but require some further extension of it with additional fields related to the higher supersymmetry). However, at tree level compactification on T^6 or $T^2 \times K3$ has a consistent truncation with $\mathcal{N} = 2$ supersymmetry that includes deformations best described by generalized complex geometry. In this paper, we will show that in this subspace of the T^6 and $T^2 \times K3$ moduli space, the black hole entropy at tree level is described by the generalized Hitchin functional. We hope that in the future examples will be found illustrating the ideas of the framework of [13, 12, 14 – 19] with $\mathcal{N} = 2$ supersymmetry.

For generalized complex space, the necessary formalism of topological strings – topological $\mathcal{J}$-model — is presented in [8], see also [20 – 27, 17, 28 – 30]. The generalized Hitchin functional in [7], and the compactifications of type II string theory on generalized CY manifolds are studied in [14, 13, 15 – 18]. For recent developments on black hole entropy see [31 – 47] and on generalized complex structures in string theory see [27, 29, 28, 48, 30, 23, 49 – 51].

In section 2 we briefly review the standard logic, in section 3 we show that it is easily generalized. In section 4 we illustrate an emergence of the generalized Hitchin functional for T^6 and $T^2 \times K3$ compactifications. The section 5 concludes the note.

2. A review

The relation between Z_{BH} and Z_{top} comes from considering a compactification of the physical type II string on a Calabi-Yau threefold X. See [52] for a comprehensive review of the subject and the complete list of references.

The resulting low energy effective theory is $\mathcal{N} = 2$ four-dimensional supergravity. It contains the $\mathcal{N} = 2$ gravitational multiplet, the universal hypermultiplet and a number of vector and hyper multiplets depending on the geometry of X. For type IIA string $h^{1,1}$ vector multiplets correspond to the complexified Kahler moduli of X, and $h^{2,1}$ hyper multiplets correspond to the complex moduli of X. For type IIB string the structure is reversed. The low energy effective action for vector multiplets is fully specified by a single holomorphic[1] function, the prepotential $\mathcal{F}(X^I)$. The X^I are scalar components of the vector multiplets, they describe moduli of the CY manifold X. The prepotential $\mathcal{F}(X)$ defines a structure of the special Kahler geometry on the corresponding moduli space.

On the one hand, the physical string amplitudes on X, which compute $\mathcal{F}(X)$, can be formulated in the language of topological strings [53, 5]. Namely, $\mathcal{F}(X)$ is just the classical free energy of the topological string. The higher genus amplitudes give the terms

$$I_g = \int d^4\theta\, W^{2g} F_g(X^I), \tag{2.1}$$

where X^k are the $\mathcal{N}=2$ chiral superfields constructed from the vector multiplets, and W is the $\mathcal{N}=2$ chiral superfield for the Weyl multiplet $W^{ij}_{\mu\nu} = T^{ij}_{\mu\nu} - R_{\mu\nu l\rho}\theta^i\sigma_{l\rho}\theta^j + \cdots$ (with T being the graviphoton field, so the expansion in components of (2.1) gives terms R^2T^{2g-2}).

On the other hand, the four-dimensional $\mathcal{N}=2$ supergravity admits BPS black hole solutions [47, 39, 54]. These BPS black hole solutions are generalizations of the extremal Reissner-Nordstrom black holes in Einstein-Maxwell theory with $M=|Q|$. The Reissner-Nordstrom black hole has the metric

$$ds^2 = -dt^2(1-2M/r+Q^2/r^2) + dr^2(1-2M/r+Q^2/r^2)^{-1} + r^2 d\Omega_2^2. \tag{2.2}$$

There is a bound $|Q|\leq M$. When the bound $M=Q$ is reached, the solution becomes BPS solution. The BPS solution preserves $N=1$ supersymmetry. Moreover, the near horizon geometry of such a solution is given by the Bertotti-Robinson metric $AdS_2\times S^2$. In coordinates, where horizon is located at $r=0$, the metric is given by

$$ds^2 = -\frac{r^2}{Q^2}dt^2 + \frac{Q^2}{r^2}(dr^2 + r^2 d\Omega^2). \tag{2.3}$$

The radius of this black hole is $r_0 = M = |Q| = |Z|$, where we also introduced the central charge of $\mathcal{N}=2$ algebra for such a BPS object. The Bekenstein-Hawking-Wald entropy [55 − 58] is given by the familiar formula

$$S = \frac{1}{4}Area = \pi r_0^2 = \pi|Q|^2. \tag{2.4}$$

In the full $\mathcal{N}=2$ supergravity we turn on the abelian vector multiplets. Each one has a complex scalar X^I and magnetic and electric fields $F^{+\,I}_{\mu\nu}, G^{+\,I}_{\mu\nu}$. In the language of special Kahler geometry, it is convenient to organize the fields into pairs $(X^I, F_I := \partial_I\mathcal{F})$ and (F^+, G^+) that transform linearly under $\mathrm{Sp}(2n+2,\mathbb{R})$ duality group, actually broken to $\mathrm{Sp}(2n+2,\mathbb{Z})$:

$$\begin{pmatrix} X^I \\ F_I \end{pmatrix} \qquad \begin{pmatrix} F^{+\,I}_{\mu\nu} \\ G^+_{\mu\nu\,I} \end{pmatrix}. \tag{2.5}$$

[1]In homogeneous special coordinates, $\mathcal{F}$ is a homogeneous function of weight 2.

The black hole can carry magnetic and electric charges (p^I, q_I), and the solution is given by the usual $\frac{1}{r^2}$ law in the metric $ds^2 = -e^{2g(r)}dt^2 + e^{2f(r)}[dr^2 + r^2 d\Omega_2^2]$

$$iF^I_{23} = i\frac{e^{-2f(r)}}{r^2}p^I, \quad iG_{23I} = i\frac{e^{-2f(r)}}{r^2}q_I. \tag{2.6}$$

It is convenient to introduce the central charge field $Z = e^{K/2}(p^I F_I - q_I X^I)$, where the Kahler potential $e^{-K} = 2\,\mathrm{Im}(X^I \bar{F}_I)$. The supersymmetry condition gives the solution for the metric in terms of Z, so we have $e^{2g(r)} = e^{-2f(r)} = e^{-K}\frac{r^2}{|Z|^2}$. From the Wald formula for the entropy one obtains again

$$S = \pi|Z|^2. \tag{2.7}$$

Since Z is expressed in terms of the scalars (X^I, F_I) and the charges (p^I, q_I) we still need to find (X^I, F_I) in terms of (p^I, q_I) and then plug into (2.7). The relation is given by the so called attractor equations[2], [60, 61, 47, 39, 62, 63]

$$\bar{Z}\begin{pmatrix} X^I \\ F_I \end{pmatrix} - Z\begin{pmatrix} \bar{X}^I \\ \bar{F}_J \end{pmatrix} = ie^{-K/2}\begin{pmatrix} p^I \\ q_J \end{pmatrix}. \tag{2.8}$$

So we have the formula for the entropy $S(p, q)$

$$S(p^I, q_I) = \pi\frac{|pF - qX|^2}{2\,\mathrm{Im}(X\bar{F})}. \tag{2.9}$$

where $\mathrm{Re}(CX^I) = p^I$, $\mathrm{Re}(CF_I) = q_I$ (we suppress index I in contractions like $X^I\bar{F}_I$). The formula (2.9) is invariant under a homogeneous complex dilatation, so we can put $C = 1$. The attractor equations $\mathrm{Re}(CX^I) = p^I, \mathrm{Re}(CF_I) = q_I$ can be also obtained minimizing (2.9) by X^I for the fixed charges (p^I, q_I) with $F_I = \partial_I\mathcal{F}$.

Let us decompose X^I into the imaginary and the real part[3] $X' + iX'', F = F' + iF''$. Then we plug $p = X', q = F'$ and compute $S(p, q) = S(X', F')$

$$S_{BH}(X', F') = \pi\frac{|X'(F' + iF'') - F'(X' + iX'')|^2}{2(X''F' - X'F'')} = \frac{\pi}{2}(X''F' - X'F''). \tag{2.10}$$

Now compare the function S_{Hit}, whose rationale will become clear in a moment, $S_{Hit}(X', F') = \frac{1}{\pi}S_{BH}(X', F')$ with the imaginary part or the prepotential $\mathcal{F} = \frac{1}{2}X^I F_I = \mathcal{F}' + i\mathcal{F}''$

$$S_{Hit}(X', F') = \frac{1}{2}(X''F' - X'F'') \tag{2.11}$$

$$\mathcal{F}''(X', X'') = \frac{1}{2}(X''F' + X'F''). \tag{2.12}$$

We see that

$$\frac{1}{\pi}S_{BH}(X', F') = S_{Hit}(X', F') = X''F' - \mathcal{F}''(X', X''). \tag{2.13}$$

[2] See [59] for studies of their relation to number theory.

[3] In the following we often omit the index I, assuming the natural contraction in products.

Moreover, since $\mathcal{F} = \frac{1}{2}X^I F_I$ and $\mathcal{F}$ is holomorphic we have the relation on the derivative

$$F_I' = \frac{\partial \mathcal{F}''}{\partial X^{I''}}. \tag{2.14}$$

Therefore $\frac{1}{\pi}S_{BH}(X', F') = S_{Hit}(X', F')$ is the Legendre transform of the imaginary part of the topological string free energy $\mathcal{F}''(X', X'')$ in the imaginary part $X'' \equiv \operatorname{Im} X$ [1, 3, 2]

$$\boxed{S_{Hit}(X', F') = \text{Legendre}[\mathcal{F}''(X', X''), F' = \partial_{X''}\mathcal{F}'']} \tag{2.15}$$

3. A generalization of the OSV conjecture

Before going to generalization, let us recall the meaning of the Hitchin functional S_{Hit} in the formulas above. Let Ω be the holomorphic $(3,0)$ form on the CY manifold X. As usual we have $X^I = \int_{A_I} \Omega, F_I = \int_{B^I} \Omega$ for some canonical basis of cycles A_I, B^I. The Hitchin functional in its critical point is the integral of the volume form defined by Ω

$$S_{Hit}(X', F') = -\frac{i}{4}\int \Omega \wedge \bar{\Omega} = \frac{1}{4i}(X\bar{F} - \bar{X}F) = \frac{1}{2}\operatorname{Im} X\bar{F}. \tag{3.1}$$

The reason why we write S_{Hit} as a function of real part of periods X', F' is that it is actually a function of them by the construction [4]

$$S_{Hit}[\rho] = \frac{1}{4i}\int (\rho + i\hat{\rho}) \wedge (\rho - i\hat{\rho}) = -\frac{1}{2}\int (\rho \wedge \hat{\rho}) = \int \sqrt{I_4(\rho)} = \int vol. \tag{3.2}$$

Here ρ is a stable real three-form, and $\hat{\rho}$ is a certain non-linear function of ρ, such that $\rho + i\hat{\rho}$ is the decomposable almost holomorphic $(3,0)$ form with respect to the complex structure also defined by ρ. The integrability of the complex structure can be cast in the form $d(\rho + i\hat{\rho}) = 0$. The field theory is defined by restricting ρ to some cohomology class in $H^3(X, \mathbb{R})$, so $d\rho = 0$. In a critical point of $S_{Hit}[\rho]$ we have $d\hat{\rho} = 0$, and the complex structure is integrable. We see that at the classical level the relation (2.15) holds: the Hitchin functional, proportional to the black hole entropy, is the Legendre transform of the imaginary part of the holomorphic prepotential $\mathcal{F}$ [1, 3, 2]. The relation between Hitchin functional and topological string was also studied classically in [6] and at the one-loop in [11]. For micro/macroscopical tests of the OSV conjecture see [31–33, 38–47].

In the case of generalized complex structures the whole construction works exactly in the same way. For a generalized complex structure, an analogue of the holomorphic $(3,0)$-form will be a mixed differential form in complex $H^{odd} = H^1 + H^3 + H^5$ or $H^{even} = H^0 + H^2 + H^4 + H^6$, which is at the same time a pure spinor $\Omega = \rho + i\hat{\rho}$ of $\mathrm{SO}(6,6)$ [7, 9]. The off-shell generalized Hitchin functional is defined by the real part ρ of the pure spinor Ω

$$S^G_{Hit} = -\frac{1}{2}\int (\rho, \hat{\rho}) = \int \sqrt{I_4(\rho)}, \tag{3.3}$$

where $\hat{\rho}$ is a certain nonlinear function of ρ, and $(,)$ is an appropriate bilinear form on the space of mixed differential forms [7]. A mixed differential form ρ in $\Omega^1 + \Omega^3 + \Omega^5$ or

$\Omega^0 + \Omega^2 + \Omega^4 + \Omega^6$, according to its chirality, transforms as a spinor of SO(6,6), and $I_4(\rho)$ is the singlet in the tensor product of four SO(6,6) spinors.

The moduli space of ordinary CY structures locally is

$$(H^{3,0} \oplus H^{2,1})(X, \mathbb{C}),$$

or $H^3(X, \mathbb{R})$ by Hitchin construction. The moduli space of generalized CY structures locally near the point of an ordinary complex structure is

$$(H^{1,0} \oplus H^{2,1} \oplus H^{3,2} \oplus H^{3,0})(X, \mathbb{C}),$$

or $H^1(X, \mathbb{R}) \oplus H^3(X, \mathbb{R}) \oplus H^5(X, \mathbb{R})$ by Hitchin construction.

The *even/odd* cases of generalized complex structure in six real dimensions correspond to the type A/B strings. They are distinguished by the chirality of the canonical pure spinor Ω that defines the corresponding generalized complex structure. In real six dimensions a usual complex structure is of odd type, and a usual symplectic structure is of even type.

In [7] Hitchin shows that the moduli space of generalized complex structures has *a special Kahler geometry*. Since $\mathcal{N} = 2$ supergravity is fully defined by an appropriate special Kahler structure on the target manifold for the scalar fields from vector multiplets, all $\mathcal{N} = 2$ computations for the black hole entropy can be done in the generalized complex case, as long as one includes the extra multiplets.

The outcome of $\mathcal{N} = 2$ supergravity is the formula (2.15), which tells us that S_{BH} is the Legendre transform of $\operatorname{Im}\mathcal{F}$. Here $\mathcal{F}$ is the prepotential of the special geometry of the moduli space of generalized complex structures. It can be defined in a similar way. We pick up a basis of A_I, B^I cycles in $H_{odd} = H_1 \oplus H_3 \oplus H_5$ or in $H_{even} = H_0 \oplus H_2 \oplus H_4 \oplus H_6$, which is canonical with respect to the *sign twisted* wedge product[4] that agrees with the bilinear form on spinors of $\mathrm{Spin}(TX, TX)$ [7]. Then

$$X^I = \int_{A_I} \Omega, \qquad F_I = \int_{B^I} \Omega, \tag{3.4}$$

where now A_I, B^I runs over all degrees in H_{odd} of H_{even}. For example, let us consider an ordinary symplectic structure ω as a generalized complex structure. Then $\Omega = e^{i\omega}$, or

$$\Omega = 1 + i\omega - \frac{1}{2}\omega^2 - \frac{1}{6}i\omega^3.$$

We have the zero-cycle and a number of two-cycles of A type, and a number four-cycles and the six-cycle of B type. The sign twisted wedge product is antisymmetric and defines a symplectic structure on $H^{even}(X)$. Then we recover the standard formulas

$$X^0 = 1 \qquad F_0 = \int_X -i\frac{1}{6}\omega^3 \tag{3.5}$$

$$X^I = \int_{A_I} i\omega \qquad F_I = \int_{B^I} -\frac{1}{2}\omega^2 \tag{3.6}$$

$$\mathcal{F} = \left(\frac{-i}{4} + \frac{i}{12}\right) \int_X \omega^3 = -\frac{i}{6} \int_X \omega^3. \tag{3.7}$$

[4]This sign twisted wedge product for two forms α, β is defined as $(\alpha, \beta) = \alpha \wedge \beta$ for $\deg \beta = 4k + 0, 1$ and $(\alpha, \beta) = -\alpha \wedge \beta$ for $\deg \beta = 4k + 2, 3$ [7].

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The topological string in a generalized complex space — topological $\mathcal{J}$-model[5] — is described in [8], see there a complete list of references on the related works. In agreement with [7] it is explained in [8], that in the case $\dim_{\mathbb{C}} X = 3$ the moduli space of geometrical deformations of a generalized complex structure is a special Kahler manifold. It is also shown that the topological string three-point function is the third derivative $C_{IJK} = \partial_I \partial_J \partial_K \mathcal{F}$ of the holomorphic prepotential $\mathcal{F}$ of that special geometry. The manifold of the geometrical deformations of a generalized complex structure is a holomorphic Lagrangian submanifold inside the total extended moduli space of deformations of the associated special differential BV algebra. The outcome of [8] is that the genus zero topological string free energy without instanton corrections is given by the same formula $\mathcal{F} = \frac{1}{2} X^I F_I$, where X_I and F^I are periods the canonical pure spinor that defines a generalized complex structure over extended set of cycles on X. Therefore the relation

$$S_{Hit}(X', F') = \text{Legendre}[\mathcal{F}''(X', X''), F' = \partial_{X''} \mathcal{F}''] \tag{3.8}$$

holds in the generalized complex case, and $\mathcal{F}''$ is the imaginary part of the free energy of the topological $\mathcal{J}$-model [8].

What about the black hole entropy? On the one hand, given a special Kahler geometry, we can formally write down an appropriate $\mathcal{N} = 2$ four-dimensional supergravity, and then the relation (2.15) automatically holds due to the special Kahler geometry relations. But can it physically be related to topological strings in generalized complex space? The answer seems to be yes, and the connection is again realized by the ten-dimensional type II string theory compactified on the given generalized CY manifold X. Recently non Calabi-Yau compactifications were studied in much details in [15, 13, 16, 18, 17]. We expect that the type II ten-dimensional string theory compactified on a generalized CY manifold X is related to the topological string on X exactly in the same fashion like in the usual case. In [12, 14] the direct relation between Hitchin functionals for generalized complex geometry in $\mathcal{N} = 2$ supergravity and the type II string compactification was described.

4. Examples: T^6 and $T^2 \times K3$

Here we consider a simple example when the generalized Hitchin functional differs from the ordinary Hitchin functional at tree level. This is possible only when X has $b_1(X) \neq 0$, so T^6 and $T^2 \times K3$ are natural examples to see explicitly how the generalized Hitchin functional works.

First of all, one shall note that the physical type II string compactified on T^6 or $T^2 \times K3$ space gives rise to $\mathcal{N} = 8$ or $\mathcal{N} = 4$ supergravity. Of course, the structure of these gravity theories differs from $\mathcal{N} = 2$. The usual, or generalized like in [8] topological string, as well as attractor equations, deals only with $\mathcal{N} = 2$ terms.

The additional massless vector multiplets of $\mathcal{N} = 4$ or $\mathcal{N} = 8$ gravities are not among observables of the topological string, which couples to variations of (generalized) complex

[5] $\mathcal{J}$ stands for a generalized complex structure, which can in particular be an ordinary symplectic (A) or an ordinary complex (B).

or symplectic structure on X. We consider $\mathcal{N} = 2$ truncation of the $\mathcal{N} = 4, 8$ theories and leave only those vector multiplets, whose scalars come from (generalized) complex or symplectic moduli of X.

In T^6 case the $\mathcal{N} = 8$ supergravity multiplet contains the following $\mathcal{N} = 2$ multiplets. There is 1 $\mathcal{N} = 2$ gravity multiplet, 6 $\mathcal{N} = 2$ gravitini multiplets, 15 $\mathcal{N} = 2$ vector multiplets and 10 $\mathcal{N} = 2$ hypermultiplets. Each gravitini multiplet has two gauge fields, so there are in total $1+12+15 = 28$ gauge fields for the T^6 compactification. We throw away the gravitini multiplets and stay with $1 + 15 = 16$ gauge fields coming from the $\mathcal{N} = 2$ supergravity sector.

In $T^2 \times K3$ case, after decomposition under $\mathcal{N} = 2$, the gauge fields are counted as follows. There is one $\mathcal{N} = 4$ supergravity multiplet and 22 $\mathcal{N} = 2$ vector multiplets. The $\mathcal{N} = 4$ supergravity multiplet is decomposed into 1 $\mathcal{N} = 2$ gravity multiplets, 2 $\mathcal{N} = 2$ gravitini multiplets and 1 $\mathcal{N} = 2$ vector multiplet. It has $1+4+1 = 6$ gauge fields. In total there are $22 + 6 = 28$ gauge fields with corresponding 28 magnetic and 28 electric charges. Again we throw away the $\mathcal{N} = 2$ gravitini multiplets and stay with $1 + 22 + 1 = 24$ gauge fields coming from $\mathcal{N} = 2$ supergravity sector.

The corresponding black hole solution carries magnetic and electric charges only for these $\mathcal{N} = 2$ vector multiplets. The solution is 1/4 BPS for T^6 compactifications and 1/2 BPS for $T^2 \times K3$, so it preserves $\mathcal{N} = 1$ four-dimensional supersymmetry. The truncation is consistent classically, and we will work here only at the classical level.

Though in derivation of the Legendre transform we closely follow [64], the novelty is the relation of the result with the generalized Hitchin functional [7] and with the generalized topological $\mathcal{J}$-model [8].

The simplest case is the 1/8 BPS black hole for the IIA on T^6 with the charges corresponding to $D0, D2, D4$ and $D6$ branes [65–67, 64, 33, 32, 31, 34]. The IIA corresponds to the topological A-model. The genus zero topological string free energy is given by

$$\mathcal{F} = -\frac{1}{6}\frac{C_{IJK}X^I X^J X^A}{X^0}, \tag{4.1}$$

where $X^I = \int_{A^I} \omega$ are integrals of the complexified Kahler class over two cycles $I = 1 \ldots 15$, and C_{IJK} is the intersection matrix for the two-cycles on T^6. We will consider A_I cycles to be the 0-cycle and all 2-cycles, the dual B^I-cycles are all 4-cycles and the 6-cycle. The 2-cycles on T^6 are labelled by pairs $1 \leq i < j \leq 6$, which we can organize into labels of the components of 6×6 antisymmetric matrix. The periods X^I, $I = 1 \ldots 15$ are entries of this matrix. The non-zero intersection of three 2-cycles correspond to the choice of three pairs of indexes (i, j) such that all of them are different, with an appropriate sign coming from parity of permutation $(i_1, i_2), (i_3, i_4), (i_5, i_6)$ into $(1, 2, 3, 4, 5, 6)$. Therefore the expression (4.1) can be reorganized into the Pfaffian of the antisymmetric 6×6 matrix X with entries X^I

$$\mathcal{F} = -\frac{\mathrm{Pf}(X)}{X^0}. \tag{4.2}$$

Now we need to find the Legendre transform of (4.2) in imaginary part $X^{I\prime\prime}$ for $X^I = X^{I\prime} + iX^{I\prime\prime}$, $I = 0 \ldots 15$. In order to do that for a general cubic prepotential of

n variables one has to solve a system of n quadratic equations, which generally speaking does not have a closed algebraic solution [68]. The key property of (4.2) that allows to explicitly find its Legendre transform in $(X^{0\prime\prime}, X^{I\prime\prime})$ is its extremely simple behavior under the full complex Legendre transform for all variables (X^I, X^0) at once. There exist a very distinguished set of cubic prepotentials $C_{IJK}X^IX^JX^K/X^0$ that are invariant under the Legendre transform in all variables. They were all algebraically classified in [69] with even stronger condition. The exponents of these functions are invariant under the Fourier transform. In the $T^2 \times K3$ case, the invariance is easy to see, and we will demonstrate it below. As for the T^6 case, see [64, 70, 69]. (The semiclassical evaluation of the Fourier transform reduces to the Legendre transform. In other words, integrals of exponents of such cubic functions are exactly localized on the their critical points.[6] Such nice prepotentials are labelled by $B_n, D_n, E_6, E_7, E_8, F_4, G_2$ algebraic types [64], and the case with Pfaffian of an antisymmetric 6×6 matrix is the E_7 case.)

Given such an invariant function $I_3(X^I)/X^0$, Pioline [64] computes its Legendre transform in $(X^{0\prime\prime}, X^{I\prime\prime})$. The idea is to shift variables $x^I = X^{I\prime\prime} - \frac{X^{0\prime\prime}}{X^{0\prime}}X^{I\prime}$ in such a way to kill the quadratic term in $X^{I\prime\prime}$ in the expansion $I_3(X^{I\prime} + iX^{I\prime\prime})$. Then the Legendre transform is computed using the invariance of $I_3(X^{I\prime\prime})/X^{0\prime\prime}$. In the notations $p = X', q = F', \phi = X''$ the Pioline result [64] is

$$\begin{aligned} S_{Hit} &= \text{Legendre}[-\operatorname{Im}\frac{I_3(p^I + i\phi^I)}{p^0 + i\phi^0}, \phi^I] = \\ &= \sqrt{4p^0 I_3(q) - 4q_0 I_3(p) + 4\partial^I I_3(q)\partial_I I_3(p) - (p^0 q_0 + p^A q_A)^2} =: \sqrt{I_4(p,q)}. \end{aligned} \tag{4.3}$$

Specializing to the T^6 case, Pioline [64] obtains quartic $\mathrm{SO}(6,6)$ invariant functional $I_4(p^I, q_I)$ of 32 charges $p^I, q_I, I = 0\ldots 16$. The charge vector p^I, q_I of T^6 transforms as a spinor under $\mathrm{SO}(6,6)$, and $I_4(p_I, q^I)$ is the singlet in the symmetric tensor product of four $\mathrm{SO}(6,6)$ spinors.

Now recall the definition (3.3) of the generalized Hitchin functional [7]. Specializing to the case of T^6, where in the critical point $\Omega = \rho + i\hat{\rho}$ is constant, one immediately recognizes the agreement with the Legendre transform (4.3) of the topological string free energy (4.2). In the framework of the generalized topological strings [8], the periods (X^I, F_I) are defined by integrals of the canonical pure spinor $\Omega = \rho + i\hat{\rho}$ of $\mathrm{SO}(TX, T^*X)$, equivalently it is a mixed differential form on X. After the Legendre transform the charges (p^I, q_I) are identified with the periods of the real part ρ of Ω. In the case of A-model, $\Omega = e^{i\omega + b}$, which gives the claimed correspondence.

What about the generalized B-model on T^6? An ordinary complex structure is defined by a holomorphic $(3,0)$ form. A generalized complex structure is defined by a pure $\mathrm{SO}(6,6)$ spinor of odd chirality, which can be represented as a mixed differential form[7] $\Omega = \Omega^{(1)} +$

[6] The same distinguished types of cubic prepotentials were also classified much earlier in [71] studies of $\mathcal{N} = 2$ supergravity.They can appear as $\mathcal{N} = 2$ four dimensional prepotentials of dimensional reduction $\mathcal{N} = 2$ five-dimensional supergravity.

[7] In this correspondence gamma matrices of $\mathrm{SO}(TX, T^*X)$ are organized into creation and annihilation operators a^{i+}, a_j, $\{a^{i+}, a_j\} = \delta^i_j$. Then $a^{i+} \simeq dx^i\wedge$ corresponds to the wedge product with dx^i, and $a_i \simeq \partial_i$ corresponds to the contraction with the vector field ∂_i.

$\Omega^{(3)} + \Omega^{(5)}$. The condition 'pure' for the SO(6,6) spinor Ω in the generalized complex case is an analogue of the (3,0) type condition for the form Ω in the ordinary complex case. Deformations of an ordinary complex structure are parameterized by Beltrami differentials $\mu^i_{\bar{j}}$, so that $\partial_{\bar{j}} \to \partial_{\bar{j}} + \mu^i_{\bar{j}} \partial_i$ and $\Omega \to e^{-\mu}\Omega$. In the generalized complex case deformations are given[8] by $\mu^{ij} + \mu^i_{\bar{j}} + \mu_{\bar{i}\bar{j}}$, which can be viewed as a section of $\Lambda^2(TX^{10} \oplus T^*X^{01}) =: \Lambda^2(L^*)$, that is a subalgebra of $so(6,6,\mathbb{C})$. A deformation $\Omega \to e^{-\mu}\Omega$ is a rotation of a spinor by an element μ of $\Lambda^2(L^*) \subset so(6,6,\mathbb{C})$. We restrict $so(6,6,\mathbb{C})$ to $\Lambda^2(L^*)$ to keep the spinor pure. Let us introduce indexes (a,b) which run over upper holomorphic ^{123} indexes and lower antiholomorphic $_{\bar{1}\bar{2}\bar{3}}$ indexes. Then an element μ_{ab} of $\Lambda^2(L^*) \subset so(6,6,\mathbb{C})$ defines a rotation of the spinor Ω by the formula

$$\Omega = e^{-\mu}\Omega_0 = e^{-\frac{1}{2}\mu_{ab}\Gamma^a\Gamma^b}\Omega_0. \tag{4.4}$$

The entries $\mu_{ab} = (\mu^{ij}, \mu^i_{\bar{j}}, \mu_{\bar{i}\bar{j}})$ are organized into an antisymmetric 6×6 matrix

$$\mu_{ab} = \begin{pmatrix} \mu^{ij} & \mu^i_{\bar{j}} \\ -(\mu^i_{\bar{j}})^T & \mu_{\bar{i}\bar{j}} \end{pmatrix}. \tag{4.5}$$

In the case of T^6 deformations, μ is a constant matrix, and the general Chern-Simons like cubic formula [8] for the tree level free energy of $\mathcal{J}$-model is reduced to

$$\mathcal{F}(\mu|\Omega_0) = -\frac{1}{6}((\mu_{ab}\Gamma^a\Gamma^b)^3\Omega_0, \Omega_0), \tag{4.6}$$

which in turn gives

$$\mathcal{F}(\mu|\Omega_0) = -\,\mathrm{Pf}(\mu). \tag{4.7}$$

We see that in the canonical coordinates, the free energy of the B-model on T^6 is also given by the cubic polynomial, namely Pfaffian of an antisymmetric 6×6 matrix. We can also write the formula in terms of periods $X^I = \int_{A_I}(\mu \cdot \Omega)$, where 15 A_I cycles in $(H_1 \oplus H_3 \oplus H_5)(X,\mathbb{C})$ are dual to the forms $\mu \cdot \Omega$ as follows. There are 3 one-cycles for dz^i, 9 three-cycles $dz^{\bar{i}} \wedge dz^j \wedge dz^k$, and 3 five-cycles $dz^1 \wedge dz^2 \wedge dz^3 \wedge dz^{\bar{i}} \wedge dz^{\bar{j}}$. In addition there is one distinguished cycle A_0, which is dual to $dz^1 \wedge dz^2 \wedge dz^3$. In terms of these periods $X_I = \int_{A^I}(\mu \cdot \Omega)$ we obtain

$$\mathcal{F} = -\frac{\mathrm{Pf}(X)}{X^0}. \tag{4.8}$$

Then one proceeds in a similar way to the A-model considered above. For an illustration let us look at the Hodge diamond of T^6. The spaces of Ω and $(\mu \cdot \Omega)$, which describe deformations of generalized complex structure with a reference point being an ordinary

[8] At an arbitrary reference point the geometrical deformations in the topological $\mathcal{J}$-model are given by $\Lambda^2(L^*)$, and all extended deformations are given by $\Lambda^\bullet(L^*)$, where L is the $+i$-eigenbundle of the generalized complex structure $\mathcal{J} \in \mathrm{End}(TX \oplus T^*X)$.

complex structure, are underlined. They are mirror to $H^0 \oplus H^2$ in the A-model by 90 degree rotation of the Hodge diamond

$$\begin{array}{ccccccc} & & & h^{00} & & & \\ & & h^{10} & & h^{01} & & \\ & h^{20} & & h^{11} & & h^{02} & \\ h^{30} & & h^{21} & & h^{12} & & h^{03} \\ & h^{31} & & h^{22} & & h^{13} & \\ & & h^{32} & & h^{23} & & \\ & & & h^{33} & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & \underline{3} & & 3 & & \\ & 3 & & 9 & & 3 & \\ \underline{1} & & \underline{9} & & 9 & & 1 \\ & 3 & & 9 & & 3 & \\ & & \underline{3} & & 3 & & \\ & & & 1 & & & \end{array}. \tag{4.9}$$

Let us remark however, that such a simple cubic formula for $\mathcal{F}$ of the generalized B-model is obtained only in the so called canonical coordinates μ, in terms of periods X^I over carefully chosen set of cycles by the condition that X^I are linear functions of μ. And the fact that $\mathcal{F}$ of the B-model on T^6 does not have corrections to the cubic term by mirror symmetry means the well-known fact that the topological A-model on T^6 does not have instanton contributions, so the formula (4.2) is exact in genus zero[9]. The function $\mathcal{F} = \frac{1}{2} X^I F_I$ is not $\mathrm{Sp}(2N)$ invariant under a change of basis of cycles, but $(X^I, F_I \equiv \partial_I \mathcal{F})$ transforms as a fundamental of $\mathrm{Sp}(2N)$. One can also compare the present computation with computation of ordinary deformations of complex structure parameterized by $H^{2,1}(T^6)$ in [59].

Let us turn to the type II string on $K3 \times T^2$ [72, 73, 65, 67, 66, 33, 31, 32, 34, 64]. As we explained above, we consider the truncation of the spectrum to $1 + 23 = 24$ gauge fields with 24 electric and 24 magnetic charges. The gauge fields come from reduction of RR $(p+1)$-forms on p-cycles on X. In type IIA p is even, and in type IIB p is odd. The Hodge diamond for $\dim H^{p,q}$ of $T^2 \times K3$ has the following form

$$\begin{array}{ccccccc} & & & h^{00} & & & \\ & & h^{10} & & h^{01} & & \\ & h^{20} & & h^{11} & & h^{02} & \\ h^{30} & & h^{21} & & h^{12} & & h^{03} \\ & h^{31} & & h^{22} & & h^{13} & \\ & & h^{32} & & h^{23} & & \\ & & & h^{33} & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & \underline{1} & & 1 & & \\ & 1 & & 21 & & 1 & \\ \underline{1} & & \underline{21} & & 21 & & 1 \\ & 1 & & 21 & & 1 & \\ & & \underline{1} & & 1 & & \\ & & & 1 & & & \end{array}. \tag{4.10}$$

Again we underlined spaces of generalized deformations with a reference point being an ordinary complex structure (the B-model). There are 22 ordinary CY moduli (20+1 for complex structures on $K3$ and T^2, and 1 for an overall dilatation of the holomorphic $(3,0)$-form) and 2 generalized extra moduli coming from deformations by a holomorphic bivector β^{ij} and B-field $B_{\bar{i}\bar{j}}$. After contraction with the holomorphic $(3,0)$ form, the β^{ij} and $B_{\bar{i}\bar{j}}$ generalized deformations sit in Ω^{10} and Ω^{32} entries of the Hodge diamond. We can decompose this deformation over the following basis in $H^{odd} = H^1 + H^3 + H^5$.

[9]Actually, the higher genus contributions also vanish.

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There is 1 deformation of complex structure on T^2, which after contraction with the holomorphic $(1,0)$ form on T^2 is mapped to the $(0,1)$ form on T^2 times the holomorphic $(2,0)$ form on $K3$. The coefficient at the form $(\overline{dz})_{T^2} \wedge \Omega_{K3}$ is called X^1.

There are 20 deformations of complex structure on $K3$ $\mu^i_{\bar{j}}$, which after contraction with the holomorphic $(2,0)$ form are mapped into $(1,1)$ forms $(\mu \cdot \Omega_{K3})_{i\bar{j}}$ on $K3$ times the holomorphic $(1,0)$ form on T^2. The corresponding coefficients are called X^I, $I = 2 \dots 21$.

There is 1 generalized deformation by holomorphic bivector on $K3$, which after contraction with the $(3,0)$ holomorphic form Ω is mapped to the $(1,0)$ holomorphic form on T^2. The corresponding coefficient is X^{22}.

There is 1 generalized deformation by $B_{\bar{i}\bar{j}}$ field, which is mapped to the space spanned by $(\overline{dz})_{T^2} \wedge \Omega_{K3} \wedge \bar{\Omega}_{K3}$. The corresponding coefficient is X^{23}.

There is 1 dilatation of Ω, which is mapped to the same space of $(3,0)$ forms Ω. The corresponding coefficient is X^0.

In total we have 24 periods X^I, $I = 0, \dots 23$ corresponding to the 24 gauge fields in $\mathcal{N} = 2$ vector multiplets.

Using formalism of [8] it is not difficult to see the topological string free energy is given by

$$\mathcal{F} = \frac{1}{2} \frac{X^1 C_{ab} X^a X^b}{X^0} \tag{4.11}$$

where C_{ab} is the intersection matrix in $H^2(K3)$, $a = 2 \dots 23$. Again the B-model answer is a simple cubic expression[10], exactly in agreement with the mirror symmetry ($T^2 \times K3$ is mirror symmetric to itself) and the fact that A-model does not have any worldsheet instanton corrections in genus zero.

The full Legendre transform of the function $\mathcal{F} = \frac{1}{2} X^1 C_{ab} X^a X^b / X^0$ in all complex variables X^I is given simply by $-\frac{1}{2} F_1 C^{ab} F_a F_b / F_0$. (In other words, for the bilinear form that satisfies $C = C^{-1}$, the function (4.11) is invariant under the full Legendre transform and fall into the classification of [69]). Explicitly, we need to solve $\partial_0 \mathcal{F} = F_0, \partial_1 \mathcal{F} = F_1$ and $\partial_a \mathcal{F} = F_a$, so we have

$$-\frac{1}{2} \frac{X^1}{(X^0)^2} C_{ab} X^a X^b = F_0 \tag{4.12}$$

$$\frac{1}{2} \frac{1}{X^0} C_{ab} X^a X^b = F_1 \tag{4.13}$$

$$\frac{X^1}{X^0} C_{ab} X^b = F_a. \tag{4.14}$$

Dividing the first line over the second, we have $\frac{X^0}{X^1} = -\frac{F_1}{F_0}$. From the third line we have $X^b = \frac{X^0}{X^1} C^{ba} F_a = -\frac{F_1}{F_0} C^{ba} F_a$. Then $C_{ab} X^a X^b = (\frac{F_1}{F_0})^2 C^{ba} F_a F_b$. Then from the second line we have $X^0 = \frac{1}{2} \frac{F_1}{(F_0)^2} C^{ba} F_a F_b$, and, using $F_0/F_1 = -X^0/X^1$ we get $X^1 = -\frac{1}{2} \frac{1}{F^0} C^{ba} F_a F_b$.

[10]The solution of the Kodaira-Spencer equation for $\bar{\partial}(a+x) + \frac{1}{2}\{(a+x),(a+x)\}$ for $\mu = x + a$, gives a nonzero correction a to the harmonic representative x of cohomology class $H^1(TX)$. However the correlation $\int_{T^2 \times K3} ((\mu \cdot)^3 \Omega, \Omega)$ decouples into $\int_{T^2}$ and $\int_{K3} ((\mu \cdot)^2 \Omega_{K3}, \Omega_{K3}) = \int_{K3} (\mu \cdot \Omega) \wedge (\mu \cdot \Omega)$. That differs from $(x \cdot \Omega) \wedge (x \cdot \Omega)$ by an integral of ∂-exact times ∂-closed term, which vanishes.

We see that X^I are expressed in terms of F_I in the same way as F_I in terms of X^I (up to the minus sign).

For any homogeneous function of weight two $\mathcal{F} = \frac{1}{2}F_I X^I$, the Legendre transform is given by $\tilde{\mathcal{F}} = F_I X^I - \mathcal{F} = \frac{1}{2}F_I X^I(F_J)$. We plug the expressions for X^I and obtain

$$\tilde{\mathcal{F}} = -\frac{1}{2}\frac{F_1 C^{ab} F_a F_b}{F_0}. \tag{4.15}$$

Then we can use Pioline [64] formula (4.3) for the Legendre transform in imaginary part of X to find

$$S_{BH} = \pi S_{Hit} = \pi\sqrt{p^2 q^2 - (p\cdot q)^2}, \tag{4.16}$$

where the charge vectors (p^I, q_I) are identified with real part of (X^I, F_I), and the scalar product is taken in the $(20,4)$ signature lattice. This is the truncation of the full $(22,6)$ lattice for type II on $T^2 \times K3$ to the charges of $\mathcal{N}=2$ vector multiplets in agreement with [72, 73, 65, 67, 33, 31, 32, 34, 64].

5. Conclusion

In this note it was argued that the OSV conjecture [1] is applicable to the case of generalized complex structures [7, 9]. If $b_1(X) = 0$ one has to use generalized Hitchin functional at quantum level [11]; classically the generalized and the ordinary geometry does not differ. However, if $b_1(X) \neq 0$, like in the case of T^6 or $T^2 \times K3$, the emergence of the generalized Hitchin functional is inevitable at tree level.

Deformations of a generalized complex structure on a three-fold X are parameterized by the half of all even/odd cycles in type A/B. The extra moduli exist at the classical level if $H^1(X)$ is not trivial. For example, the extra deformations of complex structure include $H^{1,0}$ and $H^{3,2}$ in addition to the standard $H^{2,1}$. The classical black hole entropy in this case is given by the *generalized* Hitchin functional of the form $\int\sqrt{I_4(\rho)}$, where ρ is the real part of the canonical pure SO(6,6) spinor (mixed differential form) on X. The scalar fields in $\mathcal{N}=2$ multiplets come from all such generalized deformations, and the corresponding gauge fields come from reduction of all odd/even RR forms C_p in type II A/B on all even/odd cycles in X.

We did not touch extremely interesting subjects of higher genus and nonperturbative corrections to the relation, but suggest that the generalized geometry must be an appropriate framework for study of the subject. Especially this is interesting in the context of non CY background compactifications [14, 13, 15−18]. The microscopical counting of black hole entropy could illuminate non-perturbative structure of the topological $\mathcal{J}$-model [8].

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